

# Non-Supersymmetric Unattractors in Born-Infeld Black Holes

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## Abstract

We investigate unattractor behavior in non-extremal black holes in Einstein-Born-Infeld-Dilaton theory of gravity in four-dimensional asymptotically flat spacetime. We obtain solutions which are non-singular near the horizon and dependent on the value of the dilaton field at the infinity, using perturbation method. It is shown that the value of the scalar field at the horizon is determined by its asymptotic value and the charges carried by the black hole. And we also find it is not true in general that the dilaton value at the horizon is a monotonically increasing function of the first coefficient of its series expansion in non-extremal Born-Infeld black holes.

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# 1 Introduction

In extremal black holes, the attractor mechanism states that the near horizon geometry, the field configuration of massless scalars and the black hole entropy turn out to be completely independent of the asymptotic values of radially varying scalar fields of the theory and dependent only on certain conserved quantities like mass, charges and angular momentum [1]-[11]. It was discovered first in  $N=2$  extremal black holes [8]-[11], but the concept of attractor mechanism was found to work in a much broader context and generalized beyond the original key ingredient, supersymmetry. This mechanism can be used to study the properties of extremal black holes in supersymmetric theories, or in non-supersymmetric ones. Examples of it also include black holes with higher derivative corrections, extremal black holes in higher dimensions and rotating black holes. Recently, Sen proposed, so called, the entropy function formalism [12] which is proved to be very useful in calculating the entropy of extremal black holes in a general theory of gravity, with any set of higher derivative terms and in higher dimensions [13]-[20]. This formalism is based on the facts that the near horizon geometries of the black holes are maximally symmetric and enough to give the entropy by the Wald's entropy formula, and that attractor equations are essentially some linear combinations of the equations of motion of all the fields of the theory.

Understanding the structure of the higher derivative terms is crucial because they hold a lot of information about the unitarity and renormalizability properties of the theory in question. With attractor mechanism and entropy function formalism, a lot of interesting aspects of Lovelock terms, Chern-Simons terms, Born-Infeld terms etc., can be studied [2], [21]-[23]. They are also important from the point of view of the need to introduce a small amount of non-extremality, in certain situations involving higher derivative terms [24]. So far, compared with extremal black holes, non-extremal ones are relatively less investigated. Recently, more attention has been paid to unattractor and entropy function for non-extremal black holes [1],[7],[25]-[27]. Since the Einstein field equations are a set of second order coupled nonlinear partial differential equations which are free of time derivatives in this letter, we expect that initial conditions including the first coefficient of the scalar series expansion determine all the coefficients of the field series expansion. So it is natural that non-extremal black holes show unattractor behavior. In this paper, we pay more attention to the 'degree of unattractor' and find that our naive intuition for the first coefficient of the scalar field series expansion to increase monotonically with its value at the horizon does not hold generically when Born-Infeld term is considered.

In this note, we review attractor mechanism and study unattractor in Einstein-Born-Infeld theory of gravity coupled to a massless neutral moduli field. Born-Infeld terms are known to arise in the low energy limit where gauge fields are coupled to open bosonic strings or superstrings. In fact, the low-energy effective theories of the  $D$ -brane worldvolumes are governed by Born-Infeld actions. The importance of

Born-Infeld terms on the black holes and connection with elementary string states was stressed in [28]. It was argued that virtual black holes going around closed loops can give rise to Born-Infeld type corrections to extremal black hole configurations. Einstein-Born-Infeld black holes in presence of string generated low energy fields have been studied in [29]-[34].

The rest of this paper is organized as follows. In section 2, we start with some relevant features of attractor mechanism needed for our purposes in the case of Einstein-Maxwell theory coupled to massless neutral real scalar fields. Section 3 is devoted to reviewing attractor mechanism in Einstein-Born-Infeld theory coupled to a massless neutral real scalar field, with different value of  $\lambda$  from the previous paper [2]. But we make use of a different basis. Then we investigate unattractor solutions in Einstein-Maxwell theory in section 4, and generalize them to Einstein-Born-Infeld theory in section 5, where we will investigate the degree of unattractor via the first coefficient of the scalar field series expansion. Our conclusions are summarized in section 6.

## 2 Non-Supersymmetric Attractors: General Features

Let us start with a few relevant aspects of non-supersymmetric attractors needed for our purposes. We consider the class of following gravity theories coupled to  $U(1)$  gauge fields and scalar fields as in [1]<sup>2</sup>:

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left( R - 2\partial_\mu \phi_i \partial^\mu \phi^i - f_{ab}(\phi_i) F_{\mu\nu}^a F^{b\ \mu\nu} - \frac{1}{2\sqrt{-g}} \tilde{f}_{ab}(\phi_i) F_{\mu\nu}^a F_{\rho\sigma}^b \epsilon^{\mu\nu\rho\sigma} \right) \quad (1)$$

where  $F_{\mu\nu}^a$ ,  $a = 0, \dots, N$  are gauge field strengths and  $\phi_i$ ,  $i = 1, \dots, n$  are scalar fields. Since classical non-abelian fields have never been observed, we will not take them into account in this paper. The scalar-dependent couplings of gauge fields are motivated from analogy with the supersymmetric theories. Any additional potential term for the scalar fields will lead to a breakdown of attractor mechanism in asymptotically flat spacetimes which we consider here. Rest of the notations are as in [1].

A static spherically symmetric ansatz is:

$$ds^2 = -\alpha(r)^2 dt^2 + \alpha(r)^{-2} dr^2 + \beta(r)^2 d\Omega^2 \quad (2)$$

On the other hand, the Bianchi identity and equations of motion of gauge fields can be solved by taking the gauge field strengths to be of the form:

$$F^a = f^{ab}(\phi_i) (Q_{eb} - \tilde{f}_{bc} Q_m^c) \frac{1}{\beta^2} dt \wedge dr + Q_m^a \sin \theta d\theta \wedge d\varphi, \quad (3)$$

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<sup>2</sup>In the convention of [1], the factor of  $\frac{1}{\sqrt{-g}}$  is involved in the definition of  $\epsilon^{\mu\nu\rho\sigma}$ .

where  $Q_m^a$  and  $Q_{ea}$  are constants that determine the magnetic and electric charges carried by the gauge fields  $F^a$ , and  $f^{ab}$  is inverse of  $f_{ab}$ . It is worth noting that the equations of motion (except the Hamiltonian constraint which must be imposed additionally<sup>3</sup>) can be derived from the following one-dimensional action:

$$S = \frac{1}{\kappa^2} \int dr \left[ 2 - (\alpha^2 \beta^2)'' - 2\alpha^2 \beta \beta'' - 2\alpha^2 \beta^2 \phi'^2 - \frac{2V_{eff}(\phi)}{\beta^2} \right] \quad (4)$$

with the effective potential given by

$$V_{eff}(\phi_i) = f^{ab}(Q_{ea} - \tilde{f}_{ac}Q_m^c)(Q_{eb} - \tilde{f}_{bd}Q_m^d) + f_{ab}Q_m^a Q_m^b \quad (5)$$

Now two sufficient conditions for the moduli fields to have the attractor behavior can be stated as follows [1]. First, for given charges,  $V_{eff}$  as a function of the moduli, must have a critical point  $\phi_{i0}$ . Then we have

$$\partial_i V_{eff}(\phi_{i0}) = 0 \quad (6)$$

Second, the matrix of second derivatives of the potential at the critical point,

$$M_{ij} = \partial_i \partial_j V_{eff}(\phi_{i0}) \quad (7)$$

, should have positive eigenvalues. Roughly we can write

$$M_{ij} > 0 \quad (8)$$

This condition guarantees the stability of the solutions. Once the two conditions are met, the attractor mechanism typically works [1]. As discussed in [5], it is possible that some of the eigenvalues of  $M_{ij}$  vanish. In that case the leading correction to  $V_{eff}$  along a zero mode directions should be positive for the attractor behavior to exist.

Let us concentrate on the scalar field equations of motion:

$$\partial_r(2\alpha^2 \beta^2 \partial_r \phi_i) = \frac{1}{\beta^2} \partial_i V_{eff} \quad (9)$$

Non-supersymmetric attractor equations can be derived from  $\partial_i V_{eff}(\phi_{i0}) = 0$ , which also determines the attractor values of scalar fields in terms of the fixed charges of the extremal black hole.

As discussed in [3], the above analysis can be generalized to include a certain set of higher derivative terms coming from the gravity side. In other words, it was argued in [3] that, in the presence of general  $R^2$  terms in the action, the effective

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<sup>3</sup>Otherwise, the Hamiltonian constraint can also be derived from the one-dimensional action where we replace the metric  $g_{rr}$  component  $\frac{1}{\alpha(r)^2}$  by  $\frac{1}{\eta(r)^2}$ , take the variation with respect to  $\eta(r)$  and then set  $\eta(r)$  equal to  $\alpha(r)$ .

potential gets modified by additional terms, and was in fact called as  $W_{eff}$ . The scalar field equation of motion remains as in (51), with  $V_{eff}$  replaced by  $W_{eff}$ .

Here, it should be mentioned that  $W_{eff}$  will in general depend on  $r$ . However, near the horizon all the quantities are independent of  $r$ . In this special situation the  $r$  dependence in  $W_{eff}$  drops out. As a result, the horizon radius computed from  $W_{eff}$  will also be a constant, but modified by higher derivative terms. For instance, let us note down the general form of the scalar field equation near the horizon, in the presence of Gauss-Bonnet terms:

$$(2\alpha^2\beta^2\phi')' = \frac{1}{\beta^2} \frac{dW_{eff}}{d\phi} \quad (10)$$

where

$$W_{eff}(\phi) = V_{eff}(\phi) + 4G(\phi) \quad (11)$$

and there is no  $r$  dependence. The additional term  $4G(\phi)$  also modify the entropy of the black hole via Wald's entropy formula. This is parallel to the analysis in Sen's entropy function formalism, where the addition of Gauss-Bonnet term gives rise to a finite horizon area and entropy of small black holes.

Black hole solutions in Einstein-Born-Infeld theories have been studied quite a lot in literature. It is known that one can have particle-like and BIon solutions in these theories. However, finding explicit black holes solutions in the presence of scalar couplings in the Einstein-Born-Infeld action is non-trivial. In four dimensions, when looking for asymptotically flat solutions in these theories, it is reasonable to assume that the near horizon geometry of these black holes preserve the symmetries of  $AdS_2 \times S^2$ . In order to understand the effect of higher order Born-Infeld corrections to the entropy of extremal black holes, an entropy function analysis of small black holes in heterotic string theory was presented in [23]. However, it is important to check if the attractor mechanism works when considering the full black hole solution. Later we will discuss this point in more detail. As in [1, 3], in this work, we carry out a perturbative analysis to show that the moduli fields take fixed values as they reach the horizon and that a double horizon Einstein-Born-Infeld black hole continues to exist. We show that the attractor mechanism works in the case of Born-Infeld black holes. In effect, we show that once one obtains critical values of the effective potential and ensure that  $\partial_i \partial_j V_{eff}(\phi_0) > 0$ , the perturbative analysis signifies that the attractor points remain stable.

### 3 Non-Supersymmetric Attractors in Einstein-Born-Infeld Theories

Non-supersymmetric attractor mechanism in Einstein-Born-Infeld theories can be studied using the entropy function formalism. However, to see the moduli indeed

get attracted to fixed points near the horizon, one has to use the formalism for non-supersymmetric attractor mechanism reviewed in the previous section, which make explicit use of the general solution and equations of motion [1]. In this section, we follow the analysis outlined in the previous section and the recent paper [2]. Using a perturbative approach to study the corrections to the scalar fields and taking the backreaction corrections into the metric, it is possible to show that the scalar fields are indeed drawn to their fixed values at the horizon. Here, the requirements are the existence of a *double degenerate horizon solution*. The existence of a non-supersymmetric attractor mechanism for higher derivative gravity has been recently studied in [3].

Thus, it should be interesting to use the equations of motion and study the attractor mechanism in the case of Einstein-Born-Infeld black holes coupled to moduli fields. For the purpose of studying non-supersymmetric attractor mechanism, it is instructive to start from the following action:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( R_g - 2\tilde{g}_{ij} \partial\phi^i \partial\phi^j + \mathcal{L}_{BI}^{(a)} \right) \quad (12)$$

where

$$\mathcal{L}_{BI} = 4bf_{(a)}(\phi_i) \left\{ 1 - \left[ 1 + \frac{f^{2(a)}(\phi)}{2b} F^2 - \frac{f^{4(a)}(\phi)}{16b^2} (F \star F)^2 \right]^{\frac{1}{2}} \right\}. \quad (13)$$

where we restricted to four spacetime dimensions and used the static (or Monge) gauge,  $i$  runs over the number of scalars and  $a$  is the number of gauge fields, with  $F_{\mu\nu}^a$  denoting the field strength,  $\star F$  dual to the Maxwell tensor  $F$  and  $f_{(a)}(\phi_i)$  determining the couplings. It is important not to have a potential for the scalar fields, so as to allow for a moduli space to vary. Here,  $\tilde{g}_{ij}$  stand for the metric of the moduli space. In the string theory, the parameter  $b$  is related to the inverse string tension  $\alpha'$  as  $b = (2\pi\alpha')^{-2}$ . Note that the action reduces to the Maxwell system in the  $b \rightarrow \infty$  limit. We continue to retain the Born-Infeld parameter  $b$ . In the absence of any moduli fields, Einstein-Born-Infeld black holes have been constructed in [29]. In what follows, we will be interested in asymptotically flat spacetime solutions, although the generalization to include a cosmological constant should also be possible. In fact, it might be interesting to include a cosmological constant [35] in view of the results in [23].

### 3.1 Single Scalar and Gauge Field Case

Let us take  $f_{(a)}(\phi_i) = e^{2\gamma_a \phi_i}$  where  $\gamma_a$  are parameters characterizing the coupling strength of dilaton field. It is one for string theory. We keep this parameter, for an arbitrary value of this parameter is possible in a general theory of gravity in four dimensions.

Now, one makes an ansatz for a static spherically symmetric metric which must satisfy the field equations following from the Einstein-Born-Infeld action in eqn. (12). For simplicity, we restrict ourselves to the single scalar and gauge field case<sup>4</sup>. One can generalize the result when we have several scalars and gauge fields. It should be mentioned that, although we are working with a system of gauge fields coupled to scalar fields, to lowest order, we are looking for the solution of the equations of motion only for constant values of moduli. The Birkhoff's theorem holds in this case and we may assume the solution to be static and spherically symmetric, to be of the form:

$$\begin{aligned} ds^2 &= -\alpha(r)^2 dt^2 + \frac{dr^2}{\alpha(r)^2} + \beta(r)^2 d\Omega_2^2 \\ F &= F_{tr} dt \wedge dr + F_{\theta\phi} d\theta \wedge d\phi. \end{aligned} \quad (14)$$

The induction tensor  $G_{\mu\nu}$  is defined by

$$G^{\mu\nu} = -\frac{1}{2} \frac{\partial L}{\partial F_{\mu\nu}} \quad (15)$$

The Maxwell equations and Bianchi identity are

$$dG = 0 \quad dF = 0 \quad (16)$$

These give us the following solution

$$F_{tr} = \frac{Q_e e^{2\gamma\phi}}{\beta^2 \sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi}}{b\beta^4}}}, \quad F_{\theta\phi} = Q_m \sin\theta \quad (17)$$

Although, for simplicity, we consider the case of a single scalar field, the generalization to many scalar fields is straightforward. The equations of motion and Hamiltonian constraint are driven from the action  $S$  with the above solution for gauge fields and metric ansatz, as follows

$$-1 + \left( \frac{\alpha^2 \beta^{2'}}{2} \right)' + \frac{1}{\beta^2} V_{eff} = 0 \quad (18)$$

$$\left( \frac{\alpha^{2'} \beta^2}{2} \right)' - 2b\beta^2 e^{2\gamma\phi} \left( 1 - \frac{1}{\sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi}}{b\beta^4}}} \right) = 0 \quad (19)$$

$$\partial_r (2\alpha^2 \beta^2 \partial_r \phi) - \frac{\partial_\phi V_{eff}}{\beta^2} = 0 \quad (20)$$

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<sup>4</sup>The generalization to multi-scalar fields is straightforward

$$(\partial_r \phi)^2 + \frac{\beta''}{\beta} = 0 \quad (21)$$

where  $V_{eff}$  plays a role of an ‘effective potential’ for the scalar fields. A difference with [1] is that in this case,  $V_{eff}$  is a function of  $r$ , as seen below:

$$V_{eff} = 2b\beta^4 e^{2\gamma\phi} \left( \sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi}}{b\beta^4}} - 1 \right) \quad (22)$$

However, as discussed in [4], it is possible to treat  $r$  as just a parameter near the horizon. Extremizing the effective potential gives the fixed values taken by the moduli at the horizon.

### 3.2 Perturbative Analysis

It is well known that these equations admit  $AdS_2 \times S^2$  as a solution in the case of constant moduli. However, we wish to address the attractor behavior considering double horizon black hole solutions, which are asymptotically flat. Thus, we start with an extremal black hole solution in this theory, obtained by setting the scalar fields at their critical values of the effective potential. Then, as one varies the values of scalar fields at asymptotic infinity, we show that the double horizon nature of black hole remains. Further, the critical values of the scalar fields remain stable, as the asymptotic values of these moduli fields are somewhat different from attractor values.

In view of the fact that the four equations governing  $(\alpha(r), \beta(r), \phi(r))$  are a set of four highly complicated coupled differential equations, we follow the Frobenius method to solve these equations, but exploit different basis from [3] and [2]. We call these four sets of equations of motion ( $EqA, EqB, Eq\Phi, EqC$ ). As a variable of expansion we define  $x \equiv (\frac{r}{r_H} - 1)$ . Requiring that the solution: (a) be extremal: meaning that we have double horizon<sup>5</sup>, therefore  $\alpha^2(r) = (r - r_H)^2 \tilde{\alpha}^2(r)$ , with  $\tilde{\alpha}^2(r)$  being analytic at the horizon,  $r = r_H$ , (b) be asymptotically flat: meaning that geometry and moduli tend to flat geometry at asymptotic infinity and (c) be regular at the horizon<sup>6</sup>, the most general Frobenius expansions of  $\alpha(r)$ ,  $\beta(r)$  and  $\phi(r)$  take

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<sup>5</sup>Later we will see that there is no attractor mechanism in a single-zero horizon case. Thus a double-zero horizon is important for the presence of the attractor mechanism.

<sup>6</sup>Just like the case which has been studied in [1] there is a solution where the scalar blows up at the horizon. In the supersymmetric case, the well behaved solution is automatically chosen.



the form:

$$\alpha^2(r) = \alpha_H^2 x^2 \sum_{m,n=0}^{\infty} a_{m,n} x^{m\lambda+n}, \quad (23)$$

$$\beta(r) = r_H \sum_{m,n=0}^{\infty} b_{m,n} x^{m\lambda+n}, \quad (24)$$

$$\phi(r) = \sum_{m,n=0}^{\infty} \phi_{m,n} x^{m\lambda+n}, \quad (25)$$

with  $\lambda \geq 1$ ,  $a_{0,0} = 1$ ,  $b_{0,0} = 1$  and  $\phi_{0,0} = \phi_0$ . In [1] and [2],  $\lambda$  was assumed to be very tiny positive number,  $\lambda \ll 1$ . In the above expansion, however, we assumed  $\lambda \geq 1$  for  $\partial_r \phi$  not to diverge near the horizon.

We mention that although, in comparison to (51),  $V(\phi)$  of (51) is of pure magnetic (electric) type, the case given in (51) does not have a minimum for any finite value of  $\phi$  in pure electric or magnetic case. To have a minimum in single charge case we need at least two gauge fields. Here we consider dyonic case where both electric and magnetic charges are non-zero, and assume that  $\lambda$  is slightly greater than 1 from now on.

### Zeroth order results

At zeroth order perturbation we start with a double horizon black hole solution as follows:

$$\phi(r) = \phi_0, \quad \beta(r) = r_H, \quad \alpha(r) = \alpha_H \left( \frac{r}{r_H} - 1 \right) \quad (26)$$

where for given electric and magnetic charges,  $\phi_0$ ,  $\alpha_H$  and  $r_H$ , can be found from the following equations in terms of these charges<sup>7</sup>

$$e^{4\gamma\phi_0} = \frac{Q_m^2}{Q_e^2} \alpha_H^2 \quad (27)$$

$$r_H^4 = 4Q_e^2 Q_m^2 \alpha_H^2 \quad (28)$$

$$\alpha_H^2 = 1 - \frac{1}{4bQ_m^2} \quad (29)$$

As seen above,  $\alpha_H$  is less than 1.

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<sup>7</sup>Another useful relations are as follows:

$$2br_H^2 e^{2\gamma\phi_0} = \frac{\alpha_H^2}{1 - \alpha_H^2}, \quad 1 + \frac{Q_e^2}{br_H^4} = \frac{1}{\alpha_H^2}, \quad 4bQ_m^2 = \frac{1}{1 - \alpha_H^2}$$

We should mention that from the above equations we find a lower bound for magnetic charge value  $4bQ_m^2 > 1^8$ . This bound relaxes in the limit  $b \rightarrow \infty$  where Born-Infeld theory reduces to Maxwell theory. In this limit  $\phi_0$  and  $r_H$  approach values that one can find in Einstein-Maxwell-Dilaton theory [1]. In this case, a Reissner-Nordstrom black hole with constant scalars, is an exact solution of the equations of motions.

Notice that the equations (26-29), together, determine both the attractor value of the moduli field and the horizon radius in terms of charges and the parameters of the action. In fact both the above results are meaningful. Due to (26) the Bekenstein-Hawking entropy of the solution is given by the value of the  $V_{eff}(\phi_0)$ , up to a numerical prefactor.

This in fact fixes  $\phi_0$  at its extremum point. From (25),  $\phi_0 = \phi(r_H)$  and so the value of the moduli field is fixed at the horizon, regardless of any other information. Thus to complete the proof of the attractor behavior, we should be able to show that the four sets of equations of motion, denoting a coupled system of differential equations, admit the expansions (23), (24) and (25). Furthermore, one should see that there are solutions to all orders in the  $x$ -expansion with arbitrary asymptotic values at infinity, while the value at the horizon is fixed to be  $\phi_0$ . The existence of a complete set of solutions with desired boundary conditions (considering the fact that we have coupled non-linear differential equations) by itself is not trivial. Moreover, it is easy to show that, in our theory, there is no asymptotically flat solution with everywhere constant moduli.

### First order results

To start with first order perturbation theory, we write

$$\delta\phi \equiv \phi - \phi_0, \quad (30)$$

where we keep  $\delta\phi$  as a small parameter in perturbation. From the scalar equation of motion we find

$$\delta\phi = \phi_{1,0} \left( \frac{r}{r_H} - 1 \right)^\lambda + \frac{\gamma(1 - \alpha_H^2)}{\gamma^2 - 1} \left( \frac{r}{r_H} - 1 \right) \quad (31)$$

where  $\phi_{1,0}$  is an undetermined constant and  $\lambda = \frac{1}{2}(-1 + \sqrt{1 + 8\gamma^2})$ . We see from eq. (31) that  $\delta\phi$  vanishes at the horizon and the value of the scalar is fixed at  $\phi_0$  regardless of its asymptotic value. This shows that attractor mechanism works to the first order in perturbation theory. It is noteworthy that the first order solution for the scalar field has a term with power 1 as well as general  $\lambda$ -power one. The integer-power term at the first order of the scalar field expansion appears from the

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<sup>8</sup>Note the proposal in [34] about the non-existence of the extremal limit for electrically charged black holes with Born-Infeld term.

fact that the first order term in  $\beta$ -series expansion has power 1. It really vanishes in the limit  $\alpha_H \rightarrow 1$  where Born-Infeld theory reduces to Maxwell one, as expected.

In comparison to the Einstein-Maxwell theory where an Reissner-Nordstrom black hole case was considered in [1], here we have corrections to metric components at the first order in perturbation theory. At this order,  $\alpha^2(r)$  and  $\beta(r)$  receive corrections as follows:

$$\alpha_1(r) = \alpha_H^2 a_{1,0} \left(\frac{r}{r_H} - 1\right)^{\lambda+2} + \alpha_H^2 a_{0,1} \left(\frac{r}{r_H} - 1\right)^3 \quad (32)$$

$$\beta_1(r) = r_H \left(\frac{r}{r_H} - 1\right) \quad (33)$$

where

$$a_{1,0} = \frac{4\gamma(1 - \alpha_H^2)}{(\lambda + 1)(\lambda + 2)} \phi_{1,0} \quad (34)$$

$$a_{0,1} = \frac{2(1 - \alpha_H^2)^2}{3(\gamma^2 - 1)} - 2\alpha_H^2 \quad (35)$$

This correction vanishes at the horizon faster than  $(\frac{r}{r_H} - 1)^2$ . Thus to this order, the solution continues to be a double horizon black hole with vanishing surface gravity.

## Second order results

At second order in perturbation theory the non-constant value of scalar field we found at first order, plays the role of a source. This results in corrections to the metric components. We should also consider boundary conditions as follows. Since we are interested in extremal black hole solutions with vanishing surface gravity, we should have a horizon where  $\beta(r)$  is finite and  $\alpha^2(r)$  has a "double horizon". In other words,  $\alpha(r) = (r - r_H)\tilde{\alpha}(r)$  where  $\tilde{\alpha}(r)$  is finite and non-zero at horizon. It is useful to note that, by an appropriate gauge choice, we can always take the horizon to be at  $r = r_H$ . Plugging (23-25) into equations (47-50), the solutions for  $\alpha, \beta$  and  $\phi$  corresponding to the above boundary conditions are

$$\alpha_2(r) = \alpha_H^2 a_{2,0} \left(\frac{r}{r_H} - 1\right)^{2\lambda+2} + \alpha_H^2 a_{1,1} \left(\frac{r}{r_H} - 1\right)^{\lambda+3} + \alpha_H^2 a_{0,2} \left(\frac{r}{r_H} - 1\right)^4 \quad (36)$$

$$\beta_2(r) = b_{2,0} r_H \left(\frac{r}{r_H} - 1\right)^{2\lambda} + b_{1,1} r_H \left(\frac{r}{r_H} - 1\right)^{\lambda+1} + b_{0,2} r_H \left(\frac{r}{r_H} - 1\right)^2 \quad (37)$$

$$\phi_2(r) = \phi_{2,0} \left(\frac{r}{r_H} - 1\right)^{2\lambda} + \phi_{1,1} \left(\frac{r}{r_H} - 1\right)^{\lambda+1} + \phi_{0,2} \left(\frac{r}{r_H} - 1\right)^2 \quad (38)$$

where some of the coefficients are

$$a_{2,0} = \frac{2(1 - \alpha_H^2 - \alpha_H^4 - \lambda(2\lambda + 3))}{(\lambda + 1)(2\lambda + 1)} b_{2,0} + \frac{2\gamma(1 - \alpha_H^2)}{(\lambda + 1)(2\lambda + 1)} \phi_{2,0} \quad (39)$$

$$b_{2,0} = -\frac{\lambda}{2(2\lambda-1)}\phi_{1,0}^2 \quad (40)$$

$$b_{1,1} = -\frac{\lambda(1-\alpha_H^2)}{\gamma(\gamma^2-1)}\phi_{1,0} \quad (41)$$

$$b_{0,2} = -\frac{1}{2}\left(\frac{\gamma(1-\alpha_H^2)}{\gamma^2-1}\right)^2 \quad (42)$$

$$\phi_{2,0} = \left[\frac{2\lambda(2\lambda+1)}{\gamma(\lambda+1)(\lambda+2)} - \frac{\lambda}{2\gamma(2\lambda-1)}\right](1-\alpha_H^2)\phi_{1,0}^2 \quad (43)$$

These solutions vanish at the horizon. With vanishing of  $\beta_1(r)$ , horizon area does not change to the second order in perturbation theory and is therefore independent of the asymptotic value of dilaton.  $\alpha_2(r)$  also vanishes at the horizon faster than  $\alpha_1(r)$  thus the second order solution continues to be a double horizon black hole with vanishing surface gravity.

The scalar also gets a correction to the second order in perturbation. This can be calculated in a way similar to the above analysis. We discuss this correction along with higher order corrections.

### Higher order results

We solve the system of equations ( $EqA, EqB, Eq\Phi, EqC$ ) order by order in the  $x$ -expansion. To first order, we find that one variable, say  $\phi_{1,0}$ , cannot be fixed by the equations. Let us denote the value of  $\phi_{1,0}$  as  $K$ . We thus find  $a_{1,0}$  and  $b_{1,0}$  as functions of  $K$ . One can check that at any order  $n \geq 2$ , one can substitute the resulting values of  $(a_{m,l}, b_{m,l}, c_{m,l})$ , for all  $m+l \leq n$  from the previous orders. Then ( $EqB, Eq\Phi, EqC$ ) of the current order together with  $EqA$  of order  $(n-1)$ , consistently give

$$b_{n,l} = b_{n,l}(K) \quad ; \quad a_{n,l} = a_{n,l}(K) \quad ; \quad \phi_{n,l} = \phi_{n,l}(K) \quad . \quad (44)$$

as polynomials of order  $n$  in terms of  $K$ .

$K$  remains a free parameter to all orders in the  $x$ -expansion. From (23), (24) and (25), the asymptotic values of  $(\alpha(r), \beta(r), \phi(r))$  are given by a sum of all the coefficients in the  $x$ -expansion of the corresponding function. After changing bases from  $(\frac{r}{r_H} - 1)$  to  $(1 - \frac{r}{r_H})$ , as a consequence of (44), one notices that  $(\alpha_\infty, \beta_\infty, \phi_\infty)$  are free to take different values, given different choices for  $K$ . The convergence of the series is not addressed in detail, but it would be the case for small enough values for  $|K|$ .

The arbitrary value of  $\phi$  at infinity is  $\phi = \phi_\infty$ , while its value at the horizon is fixed to be  $\phi_0$ . This signifies the presence of attractor mechanism in this theory.

## 4 Non-Supersymmetric Unattractors in Einstein-Maxwell Theories

For simplicity, we take  $f(\phi) = e^{2\gamma\phi}$  where  $\gamma$  is a parameter characterizing the coupling strength of the dilaton field. We keep this parameter, as its arbitrary value is possible in a general gravitational theory in four dimensions.

Again we assume the solution to be static and spherically symmetric, to be of the form:

$$\begin{aligned} ds^2 &= -\alpha(r)^2 dt^2 + \frac{dr^2}{\alpha(r)^2} + \beta(r)^2 d\Omega_2^2 \\ F &= F_{tr} dt \wedge dr + F_{\theta\phi} d\theta \wedge d\phi. \end{aligned} \quad (45)$$

The Maxwell equations and Bianchi identity give us the following solution

$$F_{tr} = \frac{Q_e e^{2\gamma\phi}}{\beta^2}, \quad F_{\theta\phi} = Q_m \sin \theta \quad (46)$$

The equations of motion and Hamiltonian constraint are driven from the action  $S$  with the above solution for gauge fields and metric ansatz, as follows

$$-1 + \left( \frac{\alpha^2 \beta^{2'}}{2} \right)' + \frac{1}{\beta^2} V_{eff} = 0 \quad (47)$$

$$(\alpha^2 \beta^2)'' = 2 \quad (48)$$

$$\partial_r (2\alpha^2 \beta^2 \partial_r \phi) - \frac{\partial_\phi V_{eff}}{\beta^2} = 0 \quad (49)$$

$$(\partial_r \phi)^2 + \frac{\beta''}{\beta} = 0 \quad (50)$$

where  $V_{eff}$  is an ‘effective potential’ for the scalar field. As in [1],  $V_{eff}$  in this case, is a function of  $\phi(r)$ , as seen below:

$$V_{eff} = Q_e^2 e^{2\gamma\phi} + Q_m^2 e^{-2\gamma\phi} \quad (51)$$

In this section we are considering non-extremal black holes. From the equation (44), we have  $\alpha^2 \beta^2 = (r - r_+)(r - r_-)$  where  $r_+ = r_-$  for extremal black holes and  $r_+ > r_-$  for non-extremal cases which are of interest to us at the moment.

In perturbation, at the first order, the equation for the scalar field  $\phi = \phi_0 + \delta\phi$  takes the form:

$$\partial_r (\alpha^2 \beta^2 \partial_r \delta\phi) = \frac{1}{2\beta^2} V_{eff}''(\phi_0) \delta\phi \quad (52)$$

In the vicinity of the horizon  $r = r_+$ , this is given by

$$\partial_x [x(x + 1 - \frac{r_-}{r_+}) \partial_x \delta\phi] = \frac{4\gamma^2 |Q_e Q_m|}{r_+^2} \delta\phi \quad (53)$$

Using change of variables  $x = -(1 - \frac{r_-}{r_+})y$ , we find the solution is proportional to a hypergeometric function:

$$\delta\phi = C_0 + C_1 y + C_2 y^2 + \dots \quad (54)$$

where the ellipses indicate the higher order terms in the expansion of  $\delta\phi$  around  $y = 0$ . The coefficients  $C_1, C_2, \dots$  are all determined from the equation of motion in terms of  $C_0$  which can have an arbitrary value. Here we see that  $\delta\phi$  does not vanish at the horizon, which makes  $\phi_H$  different from  $\phi_0$  in general unlike the double-horizon extremal black hole.

From now on we consider only (nonnegative-)integer-power series expansions of  $\alpha^2$ ,  $\beta$  and  $\phi$ . To solve the equations of motion, we consider the following expansions and use perturbative way.

$$\begin{aligned} \alpha^2 &= (1 - \frac{r_+}{r})(1 - \frac{r_-}{r}) + (a_2 - 1 + \frac{3r_-}{r_+})(\frac{r}{r_+} - 1)^3 + \dots \\ &= (1 - \frac{r_-}{r_+})(\frac{r}{r_+} - 1) + (\frac{2r_-}{r_+} - 1)(\frac{r}{r_+} - 1)^2 + a_2(\frac{r}{r_+} - 1)^3 + \dots \end{aligned} \quad (55)$$

$$\begin{aligned} \beta &= r + r_+ b_2 (\frac{r}{r_+} - 1)^2 + r_+ b_3 (\frac{r}{r_+} - 1)^3 + \dots \\ &= r_+ + r_+ (\frac{r}{r_+} - 1) + r_+ b_2 (\frac{r}{r_+} - 1)^2 + r_+ b_3 (\frac{r}{r_+} - 1)^3 + \dots \end{aligned} \quad (56)$$

$$\phi = \phi_H + \phi_1 (\frac{r}{r_+} - 1) + \phi_2 (\frac{r}{r_+} - 1)^2 + \dots \quad (57)$$

And we plug them into the equations of motion and find the coefficients of the power series expansions.

### First order results

$$(1 - \frac{r_-}{r_+})\phi_1 = \frac{\gamma}{r_+^2} [Q_e^2 e^{2\gamma\phi_H} - Q_m^2 e^{-2\gamma\phi_H}] \quad (58)$$

We can express reversely  $\phi_H$  in terms  $\phi_1$  and the charges, and find easily that  $\phi_H$  is a monotonically increasing function of  $\phi_1$ . For larger  $|\phi_1|$ ,  $\phi_H$  deviates further from  $\phi_0$ , as expected.

## Second order results

$$a_2 = 1 - \frac{3r_-}{r_+} + (1 - \frac{r_-}{r_+})\phi_1^2 \quad (59)$$

$$b_2 = -\frac{1}{2}\phi_1^2 \quad (60)$$

$$2(1 - \frac{r_-}{r_+})\phi_2 = -\phi_1 + \frac{\gamma}{r_+^2} [(-1 + \gamma\phi_1)Q_e^2 e^{2\gamma\phi_H} + (1 + \gamma\phi_1)Q_m^2 e^{-2\gamma\phi_H}] \quad (61)$$

## Higher order results

Using  $\alpha^2\beta^2 = (r - r_+)(r - r_-)$  and the equations (45) and (47), we can construct all the coefficients of  $\phi$ -series in terms of  $\phi_1$ , where each coefficient  $\phi_n (n \geq 2)$  is given by a polynomial of order  $n$  of  $\phi_1$  with  $\phi_1$  undetermined by the equation of motion. Then from the equation (46) all the coefficients of  $\beta$ -series can be written in terms of  $\phi_1$ , where each coefficient  $b_n (n \geq 2)$  is given by a polynomial of order  $n$  of  $\phi_1$ . All the coefficients of  $\alpha^2$ -series can appear in terms of  $\phi_1$  from the equation (44), where each coefficient  $a_n (n \geq 2)$  is a polynomial of order  $n$  of  $\phi_1$ . Finally the equation (43) can be used for consistency check. Even if we change the basis of the power series expansions from  $(\frac{r}{r_+} - 1)$  to  $(1 - \frac{r_{\pm}}{r})$  where  $\frac{r}{r_+} - 1 = \sum (1 - \frac{r_{\pm}}{r})^n$  all the coefficients would be given as polynomials of  $\phi_1$ . So if we take the limit  $r \rightarrow \infty$ , the asymptotic value  $\phi_{\infty}$  is determined by  $\phi_1$ , and vice versa<sup>9</sup>. Since  $\phi_H$  is given in terms of  $\phi_1$  and the fixed charges of the black hole, it is determined by the asymptotic value  $\phi_{\infty}$  and the charges through the above processes. Thus we observe unattractor phenomenon following this mechanism.

## 5 Non-Supersymmetric Unattractors in Einstein-Born-Infeld Theories

In this section, the Lagrangian, the metric ansatz and the equations of motion are the same to those of section 3, and the series expansions of  $\alpha^2$ ,  $\beta$  and  $\phi$  have the same form to those of section 4, but here the constant  $\alpha_H$  is defined by as follows:

$$\alpha_H^2 = 1 - 2br_+^2 e^{2\gamma\phi_H} \left( \sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}{br_+^4}} - 2 + \frac{1}{\sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}{br_+^4}}} \right) \quad (63)$$

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<sup>9</sup>After change of basis,  $\phi_{\infty}$  can be shown to be

$$\phi_{\infty} = \phi_H + \sum_{n=1}^{\infty} \sum_{m=1}^n \frac{(n-1)!}{(m-1)!(n-m)!} \phi_m \quad (62)$$

And we can get the following result from the Hamiltonian constraint at the zeroth order:

$$\alpha_H^2 \left(1 - \frac{r_-}{r_+}\right) = 1 - 2br_+^2 e^{2\gamma\phi_H} \left( \sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}{br_+^4}} - 1 \right) \quad (64)$$

Dividing these two expressions, we can compute  $(1 - \frac{r_-}{r_+})$ , and subtracting them shows that  $(1 - \frac{r_-}{r_+})$  is indeed less than 1 for non-extremal black holes.

### First order results

$$\left(1 - \frac{r_-}{r_+}\right)\phi_1 = \frac{2b\gamma r_+^2 e^{2\gamma\phi_H}}{\alpha_H^2} \left[ -1 + \frac{1 + \frac{Q_e^2}{br_+^4}}{\sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}{br_+^4}}} \right] \quad (65)$$

Reversely  $\phi_H$  can be presented by  $\phi_1$ . With the same  $V_{eff}$  as that of section 3, after taking derivative with respect to  $\phi$ , we find  $\phi_0$  is given by

$$e^{4\gamma\phi_0} = \frac{Q_m^2}{Q_e^2 \left(1 + \frac{Q_e^2}{br_+^4}\right)} \quad (66)$$

After a little tedious calculus,  $\phi_1$  can be found to monotonically increase as  $\phi_H$  increases only when  $\phi_H$  satisfies the following relation:

$$\begin{aligned} & B^2 x^3 + 2BE x^2 + 2BE x + E^2 \\ & \geq \sqrt{Bx^2 + E} (Bx^2 + 2BE x + E) \end{aligned} \quad (67)$$

with

$$B = 4bQ_m^2 \quad ; \quad E = 1 + \frac{Q_e^2}{br_+^4} \quad ; \quad x^{-1} = 2br_+^2 e^{2\gamma\phi_H} \quad . \quad (68)$$

However, in the limit  $b \rightarrow \infty$  where the theory approaches Einstein-Maxwell case, the above inequality is satisfied automatically, and such bound does not exist as in section 4, where  $\phi_H$ , the value of the scalar at the horizon, monotonically increase according to  $\phi_1$ , the first coefficient in  $\phi$ -series expansion, as consistent with our intuition.

### Second order results

$$a_2 = \frac{-4br_+^2 e^{2\gamma\phi_H}}{3\alpha_H^2 \sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}{br_+^4}}} \left[ (1 + \gamma\phi_1) \left( \sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}{br_+^4}} - 1 \right) \right]^2$$



$$-\frac{Q_e^2 + (1 + \gamma\phi_1)Q_m^2 e^{-4\gamma\phi_H}}{br_+^4 \left(1 + \frac{br_+^4}{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}\right)}] + 1 - \frac{3r_-}{r_+} + (1 - \frac{r_-}{r_+})\phi_1^2 \quad (69)$$

$$b_2 = -\frac{1}{2}\phi_1^2 \quad (70)$$

$$2(1 - \frac{r_-}{r_+})\phi_2 = -\phi_1 + \frac{2b\gamma r_+^2 e^{2\gamma\phi_H}}{\alpha_H^2} [(1 + \gamma\phi_1) \left( -1 + \frac{1 + \frac{Q_e^2}{br_+^4}}{\sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}{br_+^4}}} \right) + \frac{\frac{Q_m^2 e^{-4\gamma\phi_H}}{br_+^4} [1 + \gamma\phi_1 (1 + \frac{Q_e^2}{br_+^4})]}{\left(1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}{br_+^4}\right)^{\frac{3}{2}}} - \frac{\frac{Q_e^2}{br_+^4}}{\sqrt{1 + \frac{Q_e^2 + Q_m^2 e^{-4\gamma\phi_H}}{br_+^4}}}] \quad (71)$$

We can check that these coefficients reduce to the values of Einstein-Maxwell theories in the limit  $b \rightarrow \infty$ .

### Higher order results

From the equation (17), the coefficients  $b_n (n \geq 2)$  of  $\beta$ -series are given in terms of  $\phi_n (n \geq 1)$  by simple calculation. If we plug the power series into the rest three equations, arbitrary  $\phi_1$  determines all the other coefficients. Actually, one equation out of the three is redundant and can be used as consistency check of the results. So, in principle, the asymptotic value of the scalar field  $\phi_\infty$  can be expressed only by  $\phi_1$  and the specific charges carried by the Born-Infeld black holes, which can be evaluated after our changing basis from  $(\frac{r}{r_+} - 1)$  to  $(1 - \frac{r_+}{r})$  and taking the limit  $r \rightarrow \infty$ , and vice versa. Again the unattractor behavior is identified.

## 6 Conclusions

In this paper, we studied unattractor behavior in a theory of gravity coupled to a gauge field and a scalar field, with Born-Infeld correction in the action. By investigating solutions of the equations of motion, we observed the unattractor. We looked for possible solutions which are regular at the horizon and dependent on the asymptotic value of the scalar field. The analysis of section 4 and 5 shows the behavior for unattractor for non-extremal black holes in four-dimensional asymptotically flat spacetime. Especially, we found the fact that  $\phi_H$ , the value of the scalar at the horizon, is a monotonically increasing function of  $\phi_1$ , the first coefficient in  $\phi$ -series expansion only when  $\phi_H$  has values in certain ranges which can be determined by the derived inequality. It would be amusing to check whether this is true in black

holes with other higher derivative correction terms, and whether  $\phi_\infty$ , the value of the scalar at the infinity, is a monotonically increasing function of  $\phi_1$ .

We used a perturbative approach to study the corrections to the scalar field and take these backreaction corrections into the metric, to show that the value of the scalar field at the infinity indeed depends on the value of the scalar at the horizon, or equivalently on  $\phi_1$ . However, unlike the case of a Reissner-Nordstrom black hole, at higher orders in perturbative theory, the metric components get backreaction corrections in the Born-Infeld case. Fortunately, these corrections are small and vanish at the horizon. We showed at asymptotic infinity, there are different black hole solutions characterized by different values taken by the scalar field of the theory. Near the horizon the scalar field goes to specific values determined by its asymptotic value and the charges of the black hole. It would be interesting to generalize this analysis to the cases of asymptotic *AdS* and higher dimensional black holes.

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## References

- [1] K. Goldstein, N. Iizuka, R. P. Jena and S. P. Trivedi, “Non-supersymmetric attractors,” [hep-th/0507096].
- [2] B. Chandrasekhar, H. Yavartanoo, S. Yun, “Non-Supersymmetric Attractors in BI black holes,” [hep-th/0611240].
- [3] B. Chandrasekhar, S. Parvizi, A. Tavanfar and H. Yavartanoo, “Non-supersymmetric attractors in  $R^{*2}$  gravities,” [hep-th/0602022].
- [4] M. Alishahiha and H. Ebrahim, “Non-Supersymmetric Attractors and Entropy Function,” [hep-th/0601016].
- [5] P. K. Tripathy and S. P. Trivedi, “Non-supersymmetric attractors in string theory,” [hep-th/0511117].
- [6] S. Kar and S. Majumdar, “Noncommutative D(3)-brane, black holes and attractor mechanism,” *Phys. Rev. D* **74**, 066003 (2006) [hep-th/0606026].
- [7] J. F. Morales and H. Samtleben, “Entropy function and attractors for AdS black holes,” *JHEP* **0610**, 074 (2006) [hep-th/0608044].

- [8] S. Ferrara, R. Kallosh and A. Strominger, “N=2 extremal black holes,” Phys. Rev. D **52**, 5412 (1995) [hep-th/9508072].
- [9] A. Strominger, “Macroscopic Entropy of  $N = 2$  Extremal Black Holes,” Phys. Lett. B **383**, 39 (1996) [hep-th/9602111].
- [10] S. Ferrara and R. Kallosh, “Supersymmetry and Attractors,” Phys. Rev. D **54**, 1514 (1996) [hep-th/9602136].
- [11] S. Ferrara and R. Kallosh, “Universality of Syperymmetric Attractors,” Phys. Rev. D **54**, 1525 (1996) [hep-th/9603090].
- [12] A. Sen, “Black hole entropy function and the attractor mechanism in higher derivative gravity,” [hep-th/0506177].
- [13] T. Mohaupt, “Black hole entropy, special geometry and strings,” [hep-th/0007195].
- [14] A. Ghodsi, “R\*\*4 corrections to D1D5p black hole entropy from entropy function formalism,” [hep-th/0604106].
- [15] G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, “Stationary BPS solutions in  $N = 2$  supergravity with R\*\*2 interactions,” JHEP **0012**, 019 (2000) [hep-th/0009234].
- [16] A. Sen, “Entropy function for heterotic black holes,” [hep-th/0508042].
- [17] P. Kraus and F. Larsen, “Holographic gravitational anomalies,” JHEP **0601**, 022 (2006) [hep-th/0508218].
- [18] B. Sahoo and A. Sen, “Higher derivative corrections to non-supersymmetric extremal black holes in  $N = 2$  supergravity,” [hep-th/0603149].
- [19] G. Exirifard, “The alpha-prime stretched horizon in Heterotic string,” [hep-th/0604021].
- [20] B. Sahoo and A. Sen, “alpha’ corrections to extremal dyonic black holes in heterotic string theory,” [hep-th/0608182].
- [21] P. Prester, “Lovelock type gravity and small black holes in heterotic string theory,” [hep-th/0511306].
- [22] B. Sahoo and A. Sen, “BTZ black hole with Chern-Simons and higher derivative terms,” [hep-th/0601228].
- [23] B. Chandrasekhar, “Born-Infeld corrections to the entropy function of heterotic black holes,” [hep-th/0604028].

- [24] A. Dabholkar, A. Sen and S. Trivedi, “Black hole microstates and attractor without supersymmetry,” [hep-th/0611143].
- [25] D. Astefanesei, K. Goldstein, S. Mahapatra, “Moduli and (un)attractor black hole thermodynamics,” [hep-th/0611140].
- [26] C. M. Miller, K. Schalm, E. J. Weinberg, “Nonextremal black holes are BPS,” [hep-th/0612308].
- [27] R.-G. Cai, D.-W. Pang, “Entropy Function for Non-Extremal Black Holes in String Theory,” [hep-th/0701158].
- [28] G. W. Gibbons and D. A. Rasheed, “Electric - magnetic duality rotations in nonlinear electrodynamics,” Nucl. Phys. B **454**, 185 (1995) [hep-th/9506035].
- [29] D. L. Wiltshire, “BLACK HOLES IN STRING GENERATED GRAVITY MODELS,” Phys. Rev. D **38**, 2445 (1988).
- [30] D. A. Rasheed, “Non-linear electrodynamics: Zeroth and first laws of black hole mechanics,” [hep-th/9702087].
- [31] T. Tamaki and T. Torii, “Gravitating BIon and BIon black hole with dilaton,” Phys. Rev. D **62**, 061501 (2000) [gr-qc/0004071].
- [32] M. Wirschins, A. Sood and J. Kunz, “Non-Abelian Einstein-Born-Infeld black holes,” Phys. Rev. D **63**, 084002 (2001) [hep-th/0004130].
- [33] G. Clement and D. Gal’tsov, “Solitons and black holes in Einstein-Born-Infeld-dilaton theory,” Phys. Rev. D **62**, 124013 (2000) [hep-th/0007228].
- [34] T. Tamaki and T. Torii, “Dyonic BIon black hole in string inspired model,” Phys. Rev. D **64**, 024027 (2001) [gr-qc/0101083].
- [35] T. K. Dey, “Born-Infeld black holes in the presence of a cosmological constant,” Phys. Lett. B **595**, 484 (2004) [hep-th/0406169].